An Efficient Method for Computing the Magnetic Field Generated by Transmission Lines with Static Wires

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***Abstract* – An efficient method for computing magnetic fields generated by transmission lines is presented. The method uses a two-dimensional computation approach and gives both conductor based and phase based line parameters as well as the magnetic field. The computation results for a typical case are compared with those obtained using an independent three-dimensional analysis software and it was found that the results are in good agreement.**

I. Introduction

For a long parallel transmission line system, the magnetic field can be considered to be essentially two-dimensional. Therefore it is possible to use two-dimensional methods to compute the magnetic field.

This article presents one such method. The method accounts for the presence of both phase conductors and neutral conductors. The transmission line can have several phases and neutrals. Each phase can consist of several conductors bundled together.

The computation of the magnetic field proceeds in five steps: (1) computation of the conductor-based line parameters; (2) bundle reduction, which gives phase-based line parameters; (3) determination of the neutral current; (4) computation of the current distribution in individual conductors of the bundles; and (5) computation of the magnetic field generated by those currents..

As a practical example, a three-phase transmission line with two shield wires is studied. In this example, each phase consists of a bundle of four conductors. The effect of the size of phase wire bundle is investigated. The results were found to be essentially identical to those generated using an independent three-dimensional analysis program.

II. Computation Procedure

The computation procedure consists of the following five steps.

Step 1. Computation of the conductor based line parameters

The line parameters of a conductor include the self impedance Z_s and the mutual impedance Z_{km} given by [1],

$$Z_s = j\frac{\omega\mu_0}{2\pi}\ln\frac{2h_s}{r} + j\frac{\omega\mu_r}{\pi}J_s \tag{1}$$

$$Z_{km} = j\frac{\omega\mu_0}{2\pi}\ln\frac{\sqrt{(h_k+h_m)^2+d^2}}{\sqrt{(h_k-h_m)^2+d^2}} + j\frac{\omega\mu_r}{\pi}J_m \tag{2}$$

where

ω	angular frequency, in radians/seconds,
μ_r	relative magnetic permeability of the ground,
h_s	height of the conductor, in meters,
k, m	number of conductors,
h_k, h_m	height of the conductors *k* and *m,* in meters
d	horizontal separation between conductor *k* and *m,* in meters
J_s, J_m	earth return factors.

The expressions of J_s, and J_m are,

$$J_s = P_s + jQ_s = \int_0^\infty \frac{e^{-2h_s\lambda}}{\lambda+\sqrt{\lambda^2+j\omega\mu_r/\rho}}\,d\lambda \tag{3}$$

$$J_m = P_m + jQ_m = \int_0^\infty \frac{e^{-2(h_s+h_m)\lambda}}{\lambda+\sqrt{\lambda^2+j\omega\mu_r/\rho}}\cos(\lambda d)\,d\lambda \tag{4}$$

where ρ is the soil resistivity of the ground in Ω/m.

There are several ways to evaluate these factors. When the value of *d/h* ($h=(h_k+h_m)/2$) is within a reasonable range, (3) and (4) can be simplified to the following closed-form equations [2],

$$J_s = \frac{1}{2}\ln\left(\frac{p}{h_s}+1\right) - \frac{1}{24}\frac{1}{\left(1+\frac{h_s}{p}\right)^3} \tag{5}$$

$$J_m = \frac{1}{4}\ln\left(\frac{\left(1+\frac{p}{h}\right)^2 + \beta^2}{1+\beta^2}\right) - \frac{1}{48}\left(\frac{1}{\left(\frac{h}{p}(1+j\beta)+1\right)^3} + \frac{1}{\left(\frac{h}{p}(1-j\beta)+1\right)^3}\right) \tag{6}$$

where $p = \sqrt{j.\propto_{r} / .}$ and $\beta = d/(h_k+h_m)$.

Step 2. Apply bundle reduction

Since the current energization is based on phases, it is necessary to know the phase-based line parameters.

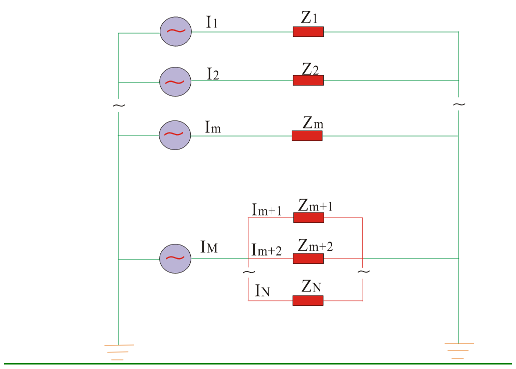


Fig. 1. Circuit of bundled conductors.

As the conductor based impedances are already computed in the previous step, the following equations can be found. For simplicity, as described in Fig.1, only neutral wires are bundled together in the derivation. Of cause, this methodology can be applied to any phase.

$$\begin{aligned}
V_1 &= Z_{11}I_1 + Z_{12}I_2 + \ldots + Z_{1m}I_m + Z_{1(m+1)}I_{1(m+1)} + \ldots + Z_{1n}I_n \\
V_2 &= Z_{21}I_1 + Z_{22}I_2 + \ldots + Z_{2m}I_m + Z_{2(m+1)}I_{2(m+1)} + \ldots + Z_{2n}I_n \\
&\ldots \\
V_m &= Z_{m1}I_1 + Z_{m2}I_2 + \ldots + Z_{mm}I_m + Z_{m(m+1)}I_{m(m+1)} + \ldots + Z_{mn}I_n \\
V_{m+1} &= Z_{(m+1)1}I_1 + Z_{(m+1)2}I_2 + \ldots + Z_{(m+1)m}I_m + Z_{(m+1)(m+1)}I_{(m+1)(m+1)} + \ldots + Z_{(m+1)n}I_n \\
&\ldots \\
V_n &= Z_{n1}I_1 + Z_{n2}I_2 + \ldots + Z_{nm}I_m + Z_{n(m+1)}I_{n(m+1)} + \ldots + Z_{nn}I_n
\end{aligned} \tag{7}$$

where V_i and I_i are the potential and current of conductor i and Z_{ij} is the mutual impedance between conductors i and j.

Note that in this and the following sections, the following symbols have been used in the derivations,

N - Total number of conductors,
M - Total Number of phases excluding neutral,
Mi - Number of conductors in *i-th* phase,
PHi - Phase number,
In - Current in *n-th* conductor,
$Itotalphi$ -Total current at *i-th* phase,

After bundle reduction, the self impedance of the neutral wire bundle is denoted as Z_{00} and the mutual impedance between the neutral and conductor j as Z_{0j}. The total current in neutral wire bundle is I_0. Rewrite the above equations as,

$$\begin{aligned}
V_1 &= Z_{11}I_1 + Z_{12}I_2 + \ldots + Z_{1m}I_m + Z_{01}I_0 \\
V_2 &= Z_{21}I_1 + Z_{22}I_2 + \ldots + Z_{2m}I_m + Z_{02}I_0 \\
&\ldots \\
V_m &= Z_{m1}I_1 + Z_{m2}I_2 + \ldots + Z_{mm}I_m + Z_{0n}I_0 \\
V_0 &= Z_{01}I_1 + Z_{02}I_2 + \ldots + Z_{0m}I_m + Z_{00}I_0
\end{aligned} \tag{8}$$

List the equations relating (7) and (8),

$$\begin{aligned}
Z_{01} &= Z_{1(m+1)}I'_{m+1} + \ldots + Z_{1n}I'_n \\
Z_{02} &= Z_{0(m+1)}I'_{m+1} + \ldots + Z_{0n}I'_n \\
&\ldots \\
Z_{0m} &= Z_{m(m+1)}I'_{m+1} + \ldots + Z_{mn}I'_n
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
Z_{00} &= Z_{(m+1)(m+1)}I'_{m+1} + \ldots + Z_{(m+1)n}I'_N \\
Z_{00} &= Z_{(m+2)(m+1)}I'_{m+1} + \ldots + Z_{(m+2)N}I'_N \\
&\ldots \\
Z_{00} &= Z_{N(m+1)}I'_{m+1} + \ldots + Z_{NN}I'_N
\end{aligned} \tag{10}$$

$$I_0 = I'_{m+1} + \ldots + I'_N$$

I'_m is the normalized neutral conductor current which is the current in each neutral conductor divided by the total neutral current, i.e.

$$I'_m = \frac{I_m}{I_0} \tag{11}$$

Solving (9) and (10) gives the impedance matrix of bundled conductors.

The above procedure can also be modified to find the symmetric components of a three phase transmission line. In order to find the symmetric components, let M=3 in (7), then apply neutral wire elimination first to get the equivalent three phase impedance matrix and finally apply the *Symmetric Components Transformation* to get the symmetric components matrix.

Step 3. Determine the current in the neutral wire bundle

After the bundle reduction for all the phases and neutral wires, the phase based impedance matrix is found. If we set the potential of the neutral wire to zero, Equation (8) can be rewritten as,

$$\begin{bmatrix} Z_{11} & \dots & Z_{1M} & Z_{10} \\ \dots & \dots & \dots & \dots \\ Z_{M1} & \dots & Z_{MM} & Z_{M0} \\ Z_{01} & \dots & Z_{0M} & Z_{00} \end{bmatrix} \bullet \begin{bmatrix} I_{ph1} \\ \dots \\ I_{phM} \\ I_0 \end{bmatrix} = \begin{bmatrix} V_{ph1} \\ \dots \\ V_{phM} \\ 0 \end{bmatrix} \quad (12)$$

Solving the last equation, the total neutral current can be found as,

$$I_0 = -\frac{\sum_{i=1}^{M} Z_{0i} I_i}{Z_{00}} \quad (13)$$

Step 4. Assign currents to individual conductors

Although the current for each phase is given and the total current in neutral wires is determined in step 3, the magnetic field cannot be computed yet because the current in each individual conductor is still unknown. In order to compute the magnetic field, the current in each conductor must be found.

The conductors in the same phase have the same voltage, so the following linear system can be set up,

$$\begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \dots & \dots & \dots & \dots \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix} \bullet \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_{ph1} \\ \vdots \\ V_{ph1} \\ V_{ph2} \\ \vdots \\ V_{ph2} \\ \vdots \\ V_0 \\ \vdots \\ V_0 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \textit{Phase } 1 \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \textit{Phase } 2 \\ \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \textit{Neutral} \end{matrix} \quad (14)$$

This system has N equations and $N+M+1$ unknowns. In order to solve this system, $M+1$ additional equations must be found. Since the total current for each phase and the neutral wire are known, the following $M+1$ equations exist,

$$\begin{aligned} & I_1 + I_2 + \dots + I_{m1} = I_{PH1} \\ & I_{m1+1} + I_{m1+2} + \dots + I_{m1+m2} = I_{PH2} \\ & \dots \\ & I_{m1+m2+..+1} + I_{m1+m2+..+2} + \dots + I_n = I_{PH(M+1)} \end{aligned} \quad (15)$$

Combining (14) and (15), the linear system can be rewritten as,

$$\begin{bmatrix} 1 & \dots & 1 & 0 & . & 0 & \dots & 0 & \dots & 0 \\ (Z_{21}-Z_{11}) & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & (Z_{2N}-Z_{1N}) \\ \vdots & \dots & \vdots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & 0 & 1 & \dots & 1 & \dots & 0 & \dots & 0 \\ (Z_{n1}-Z_{(n-1)1}) & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & (Z_{nN}-Z_{(n-1)N}) \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots \\ \vdots & \dots & \vdots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & 1 & \dots & 1 \\ (Z_{(N-m_M)1}-Z_{(N-m_M-1)1}) & \dots & \dots & \dots & \dots & \dots & \dots & \dots & & (Z_{(N-m_M)N}-Z_{(N-m_M)N}) \\ \vdots & \dots & \vdots & \vdots & \dots & \vdots & \dots & \vdots & \vdots & \vdots \end{bmatrix} \bullet \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ I_n \end{bmatrix} = \begin{bmatrix} I_{totalph1} \\ 0 \\ \vdots \\ I_{totalph2} \\ \vdots \\ \vdots \\ \vdots \\ I_{Neutral} \\ 0 \\ \vdots \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \textit{Bundle1} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \textit{Bundle2} \\ \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \textit{Neutral} \end{matrix} \quad (16)$$

Using matrix notation, equation (16) can be rewritten as,

$$[B]\bullet \vec{I}_c = \vec{I}_{ph} \quad (17)$$

Equation (17) defines matrix [B] and vector $\vec{I}_c$ which is the conductor current vector and $\vec{I}_{ph}$ which is the phase current vector. By solving this system, the currents of all the conductors can be found as,

$$\vec{I}_c = [B]^{-1} \bullet \vec{I}_{ph} \quad (18)$$

Step 5. Compute the magnetic field

The last step is to compute the magnetic field at given observation points.

The presence of the earth has been considered in the following formula to compute the magnetic field. The magnetic field *H* at observation point *j* generated by a current at position *i* is expressed as,

$$\vec{H}_{i,j} = \frac{I_i}{2\pi}\left\{\frac{\vec{\Phi}_{i,j}\hat{I}\cdot{}_i\vec{r}_{i,j}}{\left|\vec{r}_{i,j}\right|} - \frac{\vec{\Phi}'_{i,j}}{r'_{i,j}}\left[1+\frac{1}{3}\left(\frac{2}{r'_i\,.}\right)^4\right]\right\} \quad (19)$$

where

$$. = \left[j.\; u_r(\sigma + j.\; \varepsilon)\right]^{1/2}$$

$$r'_i = \left[\left(x_i - x_j\right)^2 + \left(y_i + y_j + \frac{2}{.}\right)^2\right]^{1/2}$$

$$\vec{\Phi}'_{i,j} = -\left[\frac{y_i + y_j + \frac{2}{.}}{r'_i}\right]\hat{a}_x + \frac{x_i - x_j}{r'_i}\hat{a}_y$$

where σ is the earth conductivity and ε the earth permittivity.

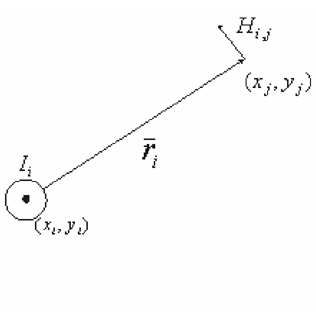


Fig. 2. Directions of magnetic field.

Fig. 2 shows the direction relations between current, distance and magnetic field vectors. The total magnetic field at observation point *j* can be obtained by adding the contributions from all the conductors.

III. Validations

The HIFREQ module of the CDEGS software package is a 3-D, full wave electromagnetic solver for computing all EM quantities for thin wire conductor systems [3]. When the length of the wire modeled in HIFREQ is long enough, the field distribution near the middle point of the length can be considered two dimensional. At this location, the results of HIFREQ and the proposed method are consistent. The advantage of the new approach is that it is much simpler to build the model and it is also faster in terms of magnetic field computation time.

IV. A Practical Example

As a practical example, consider the three-phase transmission line with two static wire bundles shown in Fig. 3. Each phase and static wire consists of a bundle of four conductors. The phase bundles are located at a height of 30 meters; the static wire bundles are at a height of 35 meters. The conductors in the bundles are located at the corners of a 1 m square. The radius of each conductor is 0.01 meter. The soil resistivity is 100 ohm-meters. The current energized in each phase is 100 Ampere, the phase angle differences are 120 degrees. The magnetic field is computed along a 100 meter long profile placed 1 meter above the earth surface and extending symmetrically on both sides of the transmission line. In order to investigate the effect of the radius of the bundles on the magnetic field, three scenarios are examined: (1) a phase consisting of a single conductor; (2) a phase consisting of a bundle of four conductors with a bundle radius of 0.707 meter; (3) a phase consisting of a bundle of four conductors with a bundle radius of 1.414 meters.

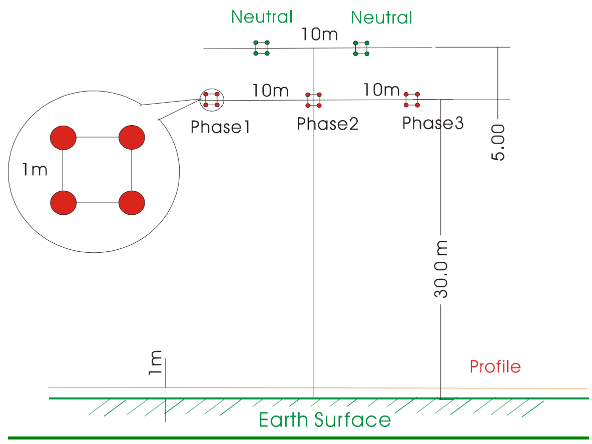
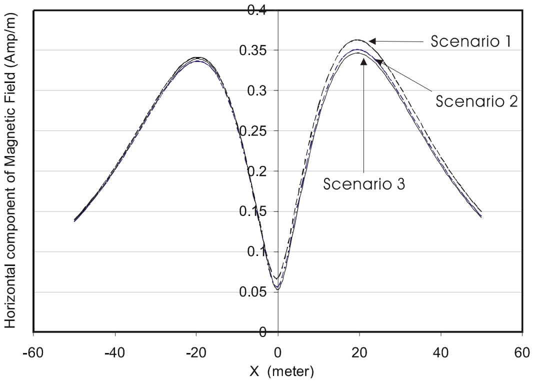
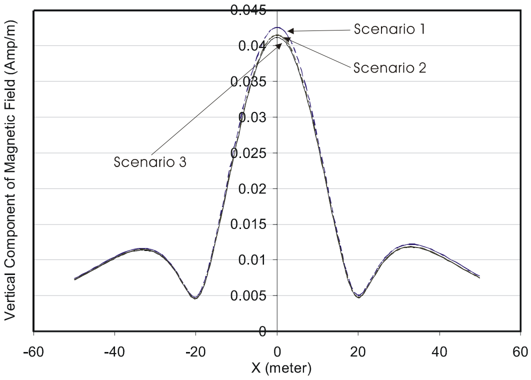


Fig. 3. Cross-section of the transmission line.



(1) Horizontal components of magnetic field



(2) Vertical components of magnetic field

Fig. 4. Computed magnitude of magnetic field.

Fig. 4 shows the computation results. It can be seen that the unbundled phase wires generate the highest magnetic field for both horizontal and vertical components. The larger bundle radius gives smaller magnetic fields. However, the effect of the bundle radius on the magnetic field is rather small. The reason is that the total current is the same when the radius of the bundle changes.

The computations were repeated with the HIFREQ program, yielding essentially identical curves.

V. References

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VI. Biographies

Dr. Yixin Yang received the B.Sc., M.Eng. and Ph.D. degrees in 1982, 1985 and 1992 respectively. From 1989 to 1997, he was a senior Electronic Engineer. From 1997 to 1998, he was a visiting fellow at Griffith University, Australia. Since September 1998, he has been with the R & D Dept. of SES in Montreal. His research interests are in transient electromagnetic scattering, EMI and EMC, and analysis of grounding systems in various soil structures.

Dr. Jinxi Ma (M'91, SM'00) was born in Shandong, P. R. China in December 1956. He received the B.Sc. degree from Shandong University, P. R. China, and the M.Sc. degree from Beijing University of Aeronautics and Astronautics, both in electrical engineering, in 1982 and 1984, respectively. He received the Ph.D. degree in electrical and computer engineering from the University of Manitoba, Winnipeg, Canada in 1991.

From 1984 to 1986, he was a faculty member with the Department of Electrical Engineering, Beijing University of Aeronautics and Astronautics. He worked on projects involving design and analysis of reflector antennas and calculations of radar cross sections of aircraft. Since September 1990, he has been with the R & D Dept. of Safe Engineering Services & technologies in Montreal, where he is presently serving as manager of the Analytical R & D Department. His research interests are in transient electromagnetic scattering, EMI and EMC, and analysis of grounding systems in various soil structures.

Dr. Ma is the author of more than eighty papers on transient electromagnetic scattering, analysis and design of reflector antennas, power system grounding, lightning, and electromagnetic interference. He is a senior member of the IEEE Power Engineering Society, a member of the IEEE Standards Association, and a corresponding member of the IEEE Substations Committee and is active on Working Groups D7 and D9.

Dr. Farid P. Dawalibi (M'72, SM'82) was born in Lebanon in November 1947. He received a Bachelor of Engineering degree from St. Joseph's University, affiliated with the University of Lyon, and the M.Sc. and Ph.D. degrees from Ecole Polytechnique of the University of Montreal.

From 1971 to 1976, he worked as a consulting engineer with the Shawinigan Engineering Company, in Montreal. He worked on numerous projects involving power system analysis and design, railway electrification studies and specialized computer software code development. In 1976, he joined Montel-Sprecher & Schuh, a manufacturer of high voltage equipment in Montreal, as Manager of Technical Services and was involved in power system design, equipment selection and testing for systems ranging from a few to several hundred kV. In 1979, he founded Safe Engineering Services & technologies, a company specializing in soil effects on power networks. Since then he has been responsible for the engineering activities of the company including the development of computer software related to power system applications.

He is the author of more than one hundred and fifty papers on power system grounding, lightning, inductive interference and electromagnetic field analysis. He has written several research reports for CEA and EPRI.

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